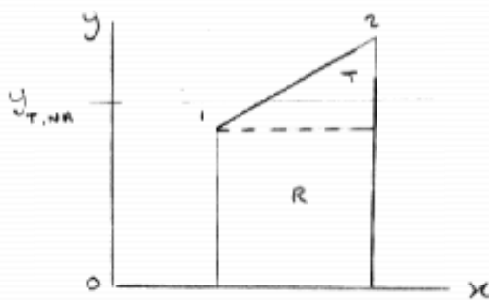


2ND Moment + Area by Noder Method

BAS007.1



$$y_{T,NA} = y_1 + (y_2 - y_1) / 3$$

$$A_T = (y_2 - y_1)(x_2 - x_1) / 2$$

$$I_{T,NA} = (x_2 - x_1)(y_2 - y_1)^3 / 36$$

T suffix: triangle
R : rectangle

$$I_{T(y=0)} = (x_2 - x_1)(y_2 - y_1)^3 / 36 + A_T \cdot y_{T,NA}^2$$

$$y_R = y_1 / 2$$

$$A_R = y_1 (x_2 - x_1)$$

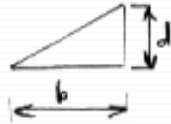
$$I_{R(y=0)} = (x_2 - x_1) y_1^3 / 3$$

$$\bar{y} = \frac{\sum (A_T y_T + A_R y_R)}{\sum (A_T + A_R)}$$

$$I_{(y=0)} = I_{T(y=0)} + I_{R(y=0)}$$

$$I_{NA} = I_{(y=0)} - \sum (A_T y_T + A_R y_R) \cdot \bar{y}$$

Principal Axes and Properties



product of inertia,
 $H_{xy} = b^2 d^2 / 72$



$$H_{xy} = 0$$

$$H_{xy} = H_{cm} + A \cdot \bar{x} \cdot \bar{y}$$

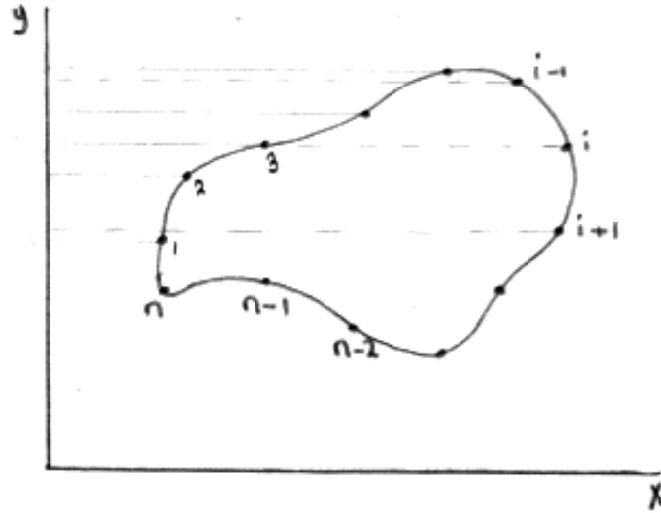
$$I_{1,2} = \frac{1}{2} (I_x + I_y) \pm \sqrt{\frac{1}{4} (I_y - I_x)^2 + H_{xy}^2}$$

$$\theta' = \frac{1}{2} \arctan \left(\frac{2 H_{xy}}{I_y - I_x} \right)$$

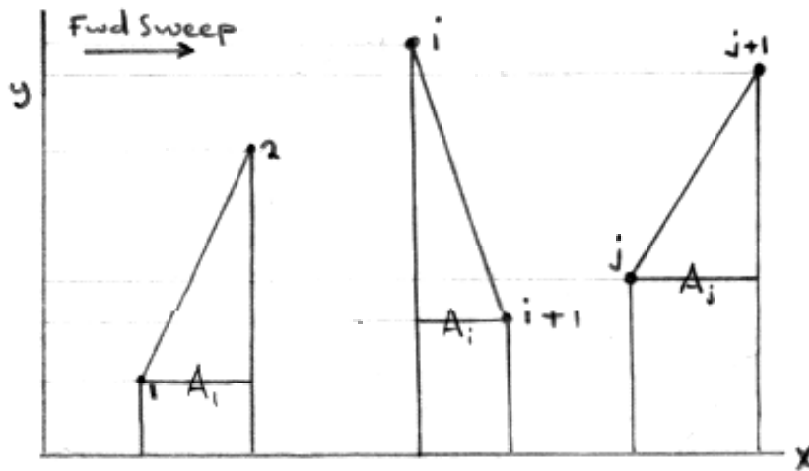
radii of giration about principal axes

$$K_{1,2} = \sqrt{I_{1,2} / A}$$

Plastic Bending

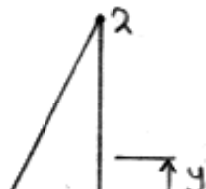


Area above assumed Zero Stress Axis



Fwd sweep ()
Bwd sweep (x)

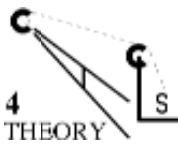
TRIANGULAR SECTION



$$y < y_1 \rightarrow A_1 = (x_2 - x_1)(y_1 - y) + ($$

$$y_2 > y > y_1 \rightarrow A_1 = (y_2 - y)^2 (x_2 - x_1) / (2$$

that is, $A_1 = (y^2 - 2y_1y + y_1^2)(x_2 - x_1)$



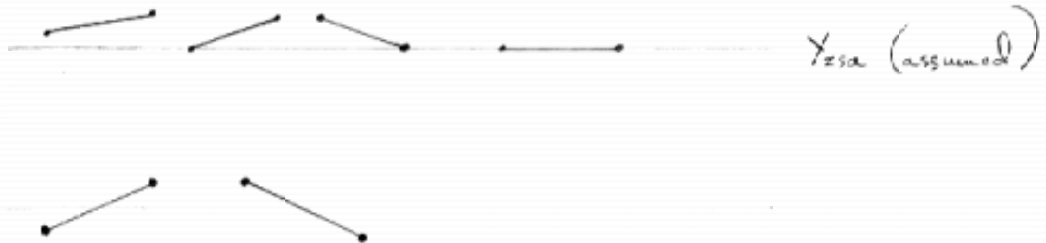
007.4

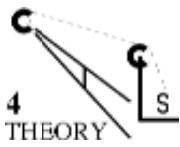
Plastic Bending 2

COEFFICIENTS OF VARIABLE y			
CONDITION	y^2	y'	y^0
$y_2 > y > y_1$	$\frac{x_2 - x_1}{2(y_2 - y_1)}$	$-2y_2 \frac{x_2 - x_1}{2(y_2 - y_1)}$	$y_2^2 \frac{x_2 - x_1}{2(y_2 - y_1)}$
$y < y_1$		$-1 \cdot (x_2 - x_1)$	$y_1(x_2 - x_1) + (x_2 - x_1)(y_2 - y_1)/2$
			$-\frac{A_T}{2}$

roots of equation

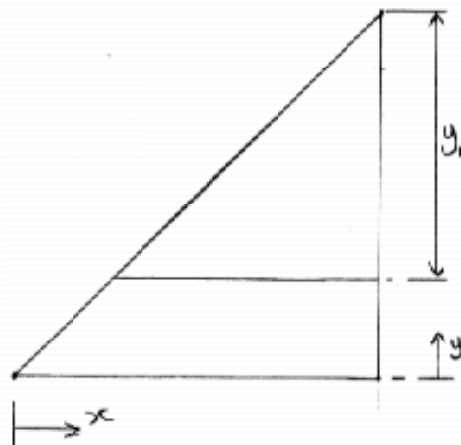
$$\text{are, } y = \frac{-b \pm (b^2 - 4ac)^{1/2}}{2a}$$





Zero Stress Axis Position for a Triangle

007.5



$$A = \frac{1}{2} x y$$

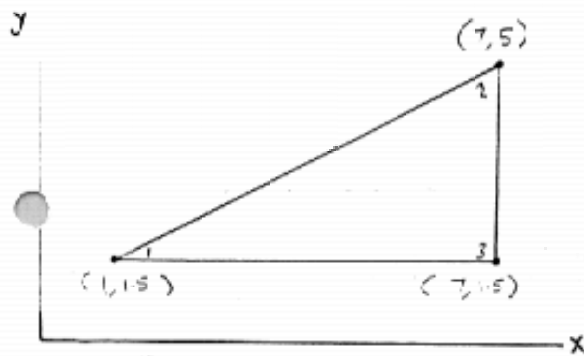
$$\frac{A}{2} = \frac{1}{4} x y$$

$$\frac{1}{4} x y = \frac{1}{2} \cdot y_1 \cdot y_1 \cdot \frac{x_1}{y_1}$$

$$y_1^2 = \frac{1}{4} y^2, \text{ then}$$

$$y_1 = y / \sqrt{2}$$

Theory Check



$$\frac{x_2 - x_1}{2(x_2 - y_1)} = \frac{(7 - 1)}{(2(5 - 1.5))} = 2.571$$

y^2	y'	y''
8571	$-2 \times 5 \times 8571$	$5^2 \times 8571$ $-(5 - 1.5)(7 - 1) / (\sqrt{2} \times 2)$

$$-8571 y^2 - 8.571 y + 16.178 = 0$$

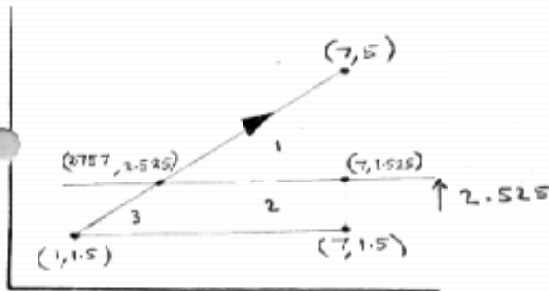
$$y = \frac{(8.571 \pm (8.571^2 - 4 \times 8571 \times 16.178)^{1/2})}{(2 \times 8571)} = 7.475$$

$$= \underline{\underline{2.525}}$$

Check

$$y_{zsa} = ((5 - 1.5) - (5 - 1.5) / \sqrt{2}) + 1.5 = 2.525 \checkmark$$

$\Sigma A.y$ check



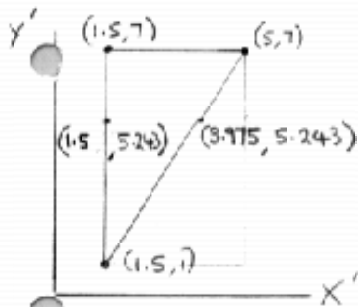
x_{zsa}	5.243	✓
Ax	12.3	✓
y_{zsa}	2.525	✓
Ay	7.176	✓

Plastic Bending
007.6

$$\begin{aligned} \Sigma A.y &= (7-2.757)(5-2.525)^2 / (2 \times 3) \\ &+ (7-2.757)(2.525-1.5)^2 / 2 \\ &+ (2.757-1)(2.525-1.5)^2 \cdot 2 / (3 \times 2) \\ &= 7.176 \end{aligned}$$

$$x_{zsa} = 1.0 + (7-1) / \sqrt{2} = 5.243$$

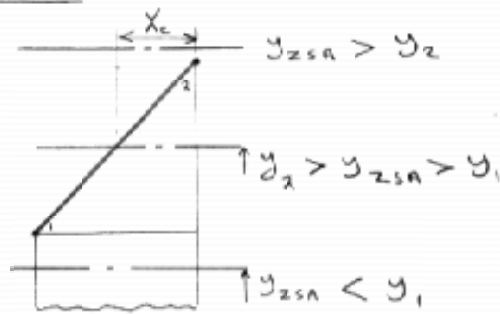
$\Sigma A.x$ check



$$\begin{aligned} \Sigma A.x &= (3.975-1.5)(5.243-1)^2 / (2 \times 3) \\ &+ (3.975-1.5)(7-5.243)^2 / 2 \\ &+ (5-3.975)(7-5.243)^2 \cdot 2 / (3 \times 2) \\ &= 12.30 \end{aligned}$$

007.7

ΣAy



$y_{zsa} < y_1$

$$A \cdot y = (x_2 - x_1) y_{zsa}^2 / 2 + (x_2 - x_1) (y_1 - y_{zsa})^2 / 2 + (x_1 - x_2) (y_2 - y_1) ((y_2 - y_1) / 3 + y_1 - y_{zsa}) / 2$$

$y_2 > y_{zsa} > y_1$

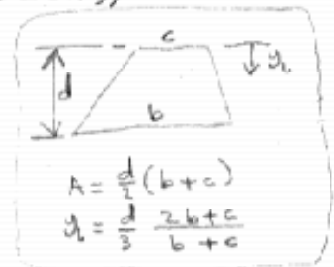
$$x_c = (y_2 - y_{zsa}) (x_2 - x_1) / (y_2 - y_1)$$

$$A \cdot y = x_c (y_2 - y_{zsa})^2 / (2 \times 3)$$

$$+ (x_2 - x_1) y_1 (y_1 / 2 + y_{zsa} - y_1)$$

$$+ ((y_{zsa} - y_1) (x_2 - x_1 + x_c) / 2)$$

$$\cdot (y_{zsa} - y_1) (2(x_2 - x_1) + x_c) / ((x_2 - x_1 + x_c)^3)$$



$y_{zsa} > y_2$

$$A \cdot y = (x_2 - x_1) y_2 (y_2 / 2 + y_{zsa} - y_2)$$

$$+ (x_2 - x_1) (y_2 - y_1) ((y_2 - y_1) / 3 + y_{zsa} - y_2) / 2$$

$y_1 > y_{zsa} > y_2$

$$x_c = (y_1 - y_{zsa}) (x_2 - x_1) / (y_2 - y_1)$$

$$A \cdot y = x_c (y_1 - y_{zsa})^2 / (2 \times 3)$$

$$+ (x_2 - x_1) y_2 (y_2 / 2 + y_{zsa} - y_2)$$

$$+ ((y_{zsa} - y_2) (x_2 - x_1 + x_c) / 2)$$

$$\cdot (y_{zsa} - y_2) (2(x_2 - x_1) + x_c) / ((x_2 - x_1 + x_c)^3)$$